647510 **MSC2MatC202x** **Seat No :________**

M.Sc. (Maths) Semester - 2 (*CBCS*) Examination
August/September -2020 [NEW COURSE]
Complex Analysis - 2 (CORE)

Time: 2:00 Hours **Marks: 56**

Instructions:

1. Figure to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any four of the following questions.

Note: Throughout G is a non-empty, open, connected subset of the complex plane C.

Q.1 Answer any seven questions. [14]

1. Let f, g : G —> C be differentiable at z_0 e G. Prove that f + g is differentiable at z_0.
2. Evaluate $/_{\backslash z \backslash = 5}$ $\cancel{(6\,2 - z^z)(z+i)}$ ^{dZ}.
3. Define entire function and give one example.
4. State the Leibnitz rule.
5. State any first version of the Maximum Modulus Theorem.
6. State second version of Cauchy's Theorem.
7. What is the Taylor's series of f (z) = sin z about zo = f ?
8. State Open Mapping Theorem.
9. State Laurent's theorem.
10. When is z_0 e C called a singularity of f ? Determine the singularities of f (z) = 1.

Q.2 Answer any two questions. [14]

(a) Let f : G —> C and g : H —> C be analytic functions such that f (G) C H. Then prove that gof is analytic on G and $(gof)'(z) = g'(f(z))f'(z)$ for all z e G.

(b) Prove that the set of all linear transformations (or Mobius transformations) forms a group under composition.

(c) Prove that a bilinear transformation is uniquely determined by its action on any three given points in C^>.

Q.3 Answer the following questions. [14]

(a) Let G be an open, connected subset of C and let f : G —> C be differentiable with f'(z) = 0 (z e G). Then prove that f is a constant function.

(b) Let G and H be open, connected subsets of C. Let f : G —> C and g : H —> C be continuous functions such that f(G) C H and $g(f$ (z)) = z (z e G). If g is differentiable and $g'(z)$ = 0 on H, then prove that f is differentiable and f'(z) = $_{g}\cancel{/(f(_z))}$.

OR

Q.3 Answer the following questions. [14]

(a) Give definitions of piecewise smooth curve and functions of bounded variation. Prove that every piecewise smooth curve is a functions of bounded variation.

(b) State and prove the Fundamental Theorem of Calculus for line integrals. [14]

Q.4 Answer any two questions.

(a) Prove the first version of Cauchy's Integral Formula.

(b) State and prove the Liouville's Theorem and the Fundamental Theorem of Algebra.

(c) State and prove Moreras Theorem.

Q.5 Answer any two questions. [14]

(a) State and prove the Schwartz Lemma.

(b) State and prove the Rouche's Theorem.

(c) State and prove the residue theorem.

(d) prove that $/_{-\circ}{}_{oo}\, j + {}_4 dx = \wedge$.

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